Applications Of Linear Algebra In Genetics

Abstract:

“I have chosen the topic “Genetics” to explore how linear algebra can be applied to this topic. More specifically, I will be focusing entirely on the phenomena of autosomal inheritance. My main goal is to show you all that how linear algebra can be used to predict the genotype distribution of a particular trait in a population after any number of generations from only the genotype distribution of the initial population. In order to perform such an analysis, I will use several critical concepts from linear algebra, including the following: Difference equations, diagonalization of a matrix, the inverse of a matrix, eigenvalues, and eigenvectors.”

Introduction:

Markov chain is a mathematical model which depends on probabilities, and is very useful in many science fields, especially in genetics. Genetics is the study of genes and is part of biology. Some problems in genetics can be solved using the Markov chain. For example, if we know the distribution of the present generation, call it, we can use the transition matrix of the population to find the genotype distribution of the future generations.

Let be the initial genotype distribution of a population. Assume that the transition matrix for the genotype of a given population is known, call it. One year later, the genotype of the population, is equal to the product of the transition matrix A and the initial matrix.

(1)

Two years later, the genotype of the population can be computed similarly,

(2)

And n years later,

(3)

According to autosomal inheritance, each inherited trait is assumed to be governed by a set of two genes. We designate these two genes by A and a. Each individual in the population under consideration has two of these genies, AA, aa or Aa.

Consider eye colouration, in humans. Genotype AA and Aa means having brown eyes and genotype aa means having blue eyes. Gene A in this case is called dominant (or we say A dominates a) and gene is called recessive (or we say a is recessive to A).

Application In Linear Algebra:

Each individual inherits one of the genes from one of its parents and another gene from its other

parent. Since each parent can pass on only one gene to the offspring, which of the two genes passes on is a matter of chance.

EXAMPLE 01: Suppose one parent is of genotype AA and the other is of type Aa. Since the offspring has two genes, one of the genes must be type A (from the first parent). The other gene can type A or type a (from the second parent). Therefore, the genotype of offspring is AA or Aa.

According to the previous technique, the table of the probability of genotype of the offspring should look like the following.

X-linked inheritance

Some genes found on the x-chromosome, are not in pair in males, female has two and male has only one. Inheritance of these genes is as follow: If offspring is a male then he receives one of his mother's two genes; if offspring is a female she receives one gene from his father and another one from her mother.

Solution: Problem 1

- a) The transition matrix A is
- b) The number of math students after one year is the first component of the vector.

Hence the number of mathematics major is 198.

- c) The number of biology students after three years is 355. It is determined by

Since

Hence there will be 355 Biology majors at senior year.

Conclusion:

We found over time, the dominant horse will outbreed the recessive horse because of the characteristics of its dominant alleles. According to our example, once the horses have bred one time, our date shows that in their offspring, the recessive gene is entirely gone. Furthermore, every time these same horses breed, the dominant gene occurs more often and the hybrid gene less among their offspring. After years of continuous breeding, the dominant gene will be present as the strongest element among these horses.

References:

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