Introduction Of The Third Truth Value In Logic

The introduction of the third truth value in logic raises some delicate questions of interpretation. In most interpretations, the third value oscillates between an ontic interpretation and an epistemic interpretation. In Lukasiewicz's original system, 'possible' was taken as 'factually unsettled'. De Finetti argues that for him propositions can only be false or true, but later he takes the third truth value to represent subjective uncertainty about the proposition and keeps the third truth value undecidable by the algorithms whether true or false. The relevant sense of epistemic needs to be qualified, because the excluded middle (A ? ¬A) remains undefined rather than true. Many systems of the three-valued logic are inclined to both interpretations. A more unbiased condition on the interpretation of the third truth value is that, depending on the application, some sentences are assigned a special semantic value, other than True or False, to present the fact that such sentences are not arguable in the way True and False sentences are, and that they do not necessarily support the same inferences. In theories of presupposition projection, the third truth value is used to represent cases in which a sentence is concluded inappropriate, for cases of presuppositional failure. The value ½, here, stands for inappropriate or defective. In some theories of Vagueness, the ½ value is used to assign a special semantic status of borderline cases and again the choice of the scheme will depend on how the vagueness of a subsentence is inherited to larger sentences. If someone considers Bochvar's treatment of paradoxical sentences, then the third value will come out to be meaningless to separate a class of sentences from true or false values. The same concern is at stake in Kripke's theory of truth, where Kleene's three-valued logic is used to lay out sentences such as the Liar or the Truth-Teller, which Kripke defines as 'ungrounded'.

From the logical point of view, the three-valued logic can be seen as only a ?rst step towards the broader family of many-valued logic. Sometimes, the three-valued logic is viewed as offering only a limited excess of freedom over two-valued logic for that matter, mostly by increasing the space of interpretations for the logical connectives, that is the option of 39 truth-functional tables vs 24 choices in a two-valued logic for a binary connective. This seems to be a significant rise in the possibilities, especially when it comes to the modeling of subtle connectives like the conditionals. But depending on the applications it is sometimes considered desirable to introduce even more truth values. On closer examination, the three-valued logic offers more options. As already mentioned, the elucidation of connectives in three-value settings need not be truth-functional, the third value can be assigned non-truth functionally. And in another way, the de?nition of logical consequence in a three-valued system leaves various choices open. When logical results are interpreted in terms of the preservation of the designated values, then one basic choice is between the preservation of the value (1), that is the preservation of Truth or strong consequence, and other is the preservation of the non-zero values(1, ½), that is the preservation of nonFalsity or weak consequence.

In the case of three-valued conditional logics, many combinations of the choices have also been considered. For example, it is standard to require both directions of the preservation simultaneously which corresponds to intersecting the logic, in the case of conditionals. And of course, there will be another possibility. We can define results in a mixed way: when all premises take the value 1, not all conclusions should take the value 0; or dually, when no premise takes the value 0, some conclusion takes the value 1.

This perspective of logical consequence has been encouraged and taken further in recent years in relation to the treatment of the paradoxes of vagueness, as well as the semantic paradoxes. Such developments have opened new logical perspectives, in particular regarding the interest of nontransitive logics toward a uni?ed treatment of the paradoxes of the vagueness and of the self-referential truth. On the technical side, further recent explorations of the three-valued systems have troubled the proof theory of three-valued logic. The paradigmatic logic of Lukasiewicz, Kleene, and De Finetti all share a common base, which stressed the interpretation of negation, disjunction, and conjunction, but they differ in systematic ways on the clarification of the conditional. The number of variants of the systems is too large, but the relation between these logics and the search of integrated proof systems for them has been a fact of continued interest. Some recent interest has also concerned the relation between three-valued logics and modal logics. From the early days of many-valued logic, it has been known that normal modal operators are not appropriately expressible by means of n-valued truth-functional connectives, which clearly shows that modal logics are more expensive than the n-valued logics, can also say the three-valued logics. But all these discussions arise a natural question to us, whether many-valued systems, and in particular the three-valued systems, can be planted in a two-valued modal system?